Homework #2

Drunkard random walk

A drunken person is attempting to navigate his way home along a street. His movement is dictated by the flip of two fair coins (coin 1 & 2) and one additional fair coin (coin 3): two tails from **coin 1 and 2** move him one step backward (west), two heads propel him one step forward (east), and a combination of a head and a tail keeps him stationary. A head from **coin 3** moves him north, while a tail moves him south. He must progress 10 steps forward and 5 steps south to reach his home.

In this assignment, you will write a blog post to explain how you used computer simulation to understand the operating characteristics of the above strategy.

- The audience of your blog post is a sharp college freshman with little to no background in data science.
- Set the maximum number of flips at 5,000, meaning that the person fails to return to his home if he cannot progress 10 steps forward and 5 steps south in 5,000 flips.
- You should explain how you used computer simulation to calculate the average number of flips to return to his home, conditioning on his being able to progress 10 steps forward and 5 steps south. As part of the explanation, provide a figure (or a series of figures) that show how the drunkard moves after each flipping the three fair coins. The x-axis represents the west-east direction, and the y-axis represents the south-north direction. Note that one flip indicates flipping the three coins.
- Now suppose that **coin 1** is biased toward a head, that is $Pr(head) = 0.75$, whereas the other two (**coin 2 and 3**) are fair coins. Please explain how this change impacts on the average number of flips to return to his home.
- Suppose that **coin 1 and 2** are biased toward a head, that is $Pr(head) = 0.75$. Please explain how this change impacts on the average number of flips to return to his home.
- Please set your simulation seed at 1 and perform simulation 5,000 times. When you compute the average number of flips, please condition on the cases that the person returns to home.

```r
# step 1: set your initial parameters

set.seed(1)  # set seed at 1
m = 5000     # number of simulation = 5000 or larger

# step 2.1 : function to simulate two coin flips and decide to move forward, backward, or stay
# step 2.2 : function to simulate one coin flips and decide to move north or south

# step 3: function to compute the number of coin flips to progress 10 steps and 5 steps south


# step 4: Repeat step 2 and 3 5000 times (i.e., m = 5000)
```

Submission instructions

- Create a folder called 02-randomwalk-simulation
- Within the folder, create a .pdf or .html file with the name writeup. Please include your **.rmd** file in the folder and then zip the folder.

- Submit the zipped folder via **Brightspace** when the assignment is due.
- The solution should be your own work. You may discuss concepts with classmates, but you may **not** share code or text.

A drunken person is attempting to navigate his way home along a street. His movement is dictated by the flip of two fair coins (coin 1 & 2) and one additional fair coin (coin 3): two tails from **coin 1 and 2** move him one step backward (west), two heads propel him one step forward (east), and a combination of a head and a tail keeps him stationary. A head from **coin 3** moves him north, while a tail moves him south. He must progress 10 steps forward and 5 steps south to reach his home.

Section 1 — Baseline: all three coins fair

Coin 1 and coin 2 together determine east-west movement each flip; coin 3 alone determines north-south movement.

$P(A \cap B) = P(A) \cdot P(B)$

Needs 10 east forward

5 south

EAST-WEST LEG (coins 1 & 2)

1.1. List all 4 equally-likely outcomes of (coin 1, coin 2): HH, HT, TH, TT.

$P(H \text{ and } H) = .5 \cdot .5 = .25$

$P(H \text{ and } T) = .5 \cdot .5$

$P(T \text{ and } H) = .25 = P(H \text{ and } T)$

$P(T \text{ and } T) = .5 \cdot .5 = .25$

1.2. Calculate P(East) — the probability both coins land heads.

$P(H \text{ and } H) = .5 \cdot .5 = .25$

Forward

1.3. Calculate P(West) — the probability both coins land tails.

$P(T \text{ and } T) = .5 \cdot .5 = .25$

Back

1.4. Calculate P(Stay) — probability of exactly one head and one tail (either order). Check that P(East) + P(West) + P(Stay) = 1.

$P(H \cap T) = .5 \cdot .5 = .25$

$P(T \cap H) = .5 \cdot .5 = .25$

$P(\text{Stay}) = P(T \cap H) + P(H \cap T) = .50$

NORTH-SOUTH LEG (coin 3)

1.5. Calculate P(North) and P(South) directly from a single fair coin.

$P(\text{North}) = P(H) = .5$

$P(\text{South}) = P(T) = .5$

EXPECTED MOVEMENT PER FLIP

Define ∆X as east-west displacement in one flip (+1 east, –1 west, 0 stay), and ∆Y as south-north displacement (+1 south — the direction toward home — –1 north).

E[∆X] = (1)·P(East) + (–1)·P(West) + (0)·P(Stay)

.25 + –.25 + 0

E[∆Y] = (1)·P(South) + (–1)·P(North)

.50 + –.50

1.6. Compute E[∆X] and E[∆Y] for this all-fair case.

E[∆X] and E[∆Y] = 0 & 0 = 0

1.7. In a sentence: what does it mean for this random walk that its expected per-flip displacement is 0? (This matters for how long it takes to drift 10 east and 5 south.)

because E[∆X] = E[∆Y] = 0, any progress towards home is random fluctuation (the variance around the zero average).

Because there is no push it will take some time for the drunkard to get home.

Section 2 — Scenario A: coin 1 biased, coins 2 & 3 fair

Coin 1 now has Pr(heads) = 0.75. Coin 2 and coin 3 remain fair (Pr(heads) = 0.5).

2.1. Recompute P(East) = Pr(coin 1 = H) · Pr(coin 2 = H) using the new coin 1 probability.

$P(East) = .75 \cdot .50 = .375$

2.2. Recompute P(West) = Pr(coin 1 = T) · Pr(coin 2 = T).

$P(West) = .25 \cdot .50 = .125$

2.3. Recompute P(Stay) = 1 – P(East) – P(West). Verify it also equals Pr(H,T) + Pr(T,H) under the new probabilities.

$P(Stay) = 1 - .375 - .125 = .5$

$P(H \cap T) = \boxed{.75} \cdot \boxed{.50} = .375$

$P(T \cap H) = \boxed{.25} \cdot \boxed{.50} = + .125$

$\} .5$

Coin #1 Coin #2

2.4. Recompute E[ΔX] with these updated probabilities.

$E[\Delta X] = .375 + -.125 + 0 = .25$

2.5. Do P(North) and P(South) change in this scenario? Explain why or why not in one sentence.

no because the pr of coin 3 is 50/50.

2.6. Prediction: based on the sign and size of your new E[ΔX], will the average number of flips to reach home go up or down versus the all-fair case? Justify it in one sentence — write this before you simulate anything.

Because $E[\Delta X] = .25$ is measuring the average eastward steps per flip meaning it can be reached in less time because it is greater than zero.

P(East) .25/.375/.5265

Section 3 — Scenario B: coins 1 & 2 both biased, coin 3 fair

Both coin 1 and coin 2 now have Pr(heads) = 0.75. Coin 3 stays fair.

3.1. Recompute P(East), P(West), and P(Stay) with both coin probabilities updated.

P(East) = P(H ∩ H) = .75 • .75 = .5625 +

P(West) = P(T ∩ T) = .25 • .25 = .0625 +

P(Stay) = P(H ∩ T) = .75 • .25 = .1875
= P(T ∩ H) = .25 • .75 = .1875 } .375 =

1

3.2. Recompute E[ΔX] for this scenario.

P(H∩H) + P(T∩T) + stay
E[ΔX] = .5625 + -.0625 + 0 = .5

3.3. Compare this E[ΔX] to Scenario A's. Is the increase in drift proportional to the single-coin change, or larger/smaller? (Think about how two independent probabilities combine — multiplicatively, not additively.)

all fair E[ΔX]

Scenario A = E[ΔX] = .375 + -.125 + 0 = .25 - 0 = .25
Scenario B = E[ΔX] = .5625 + -.0625 + 0 = .5 - .25 = .25

It is proportional

3.4. Prediction: how should the average number of flips to reach home in Scenario B compare to both the all-fair case and Scenario A? Write your prediction before simulating.

The average number of flips needed to make it home declines as E[ΔX] changes based on coin probability with all-fair needing the most flips and Scenario B needing the least

Section 4 — Using these numbers to check your simulation

Once your R simulation runs, come back to these hand calculations as a real check, not a formality.

4.1. Tabulate the empirical proportion of East / West / Stay outcomes across all simulated flips in each scenario. Compare to your hand-calculated P(East), P(West), P(Stay) — note how close they are.

Scenario		West (-1)	Stay (0)	East (1)
All fair	Predicted	.25	.50	.25
	Simulated	.25202	.49764	.25034
A	Predicted	.125	.50	.375
	Simulated	.12481	.50064	.37453
B	Predicted	.0625	.375	.5625
	Simulated	.06240	.3745	.56325

4.2. Confirm the simulated average number of flips to reach home moves in the direction your E[ΔX] predicted (down for Scenario A, further down for Scenario B) relative to the all-fair case.

$E[\Delta X] = (\overset{\text{Fair}}{0} \to \overset{\text{ScA}}{.25} \to \overset{\text{ScB}}{.5})$ Predicted

Actual = (1147.8 → 42.8 → 21.5)

as predicted, the number of draws decreases approx 0 → .25 → .5 in each scenario

4.3. Confirm the north-south behavior of the walk looks essentially unchanged across all three scenarios, since coin 3 is never altered. If it changes noticeably, that points to a bug rather than a real effect.

It is unchanged because it's probability never changes. It is .5 throughout every draw.

Keep your answers here next to you while writing the blog post — the strongest writeups state what was expected before simulating, then show whether the simulation agreed.